the intensity of noise generated by this mechanism and has not attempted to solve the problem of the transmission of the noise out of the inlet duct, which is of equal importance. It has been shown in the case of potential-flow variations that the upstream row generated as much noise as the following row. As the leading row in a compressor is in a preferential position from the propagation point of view, this may be the major source of noise heard outside the inlet duct. When wakes become appreciable at high incidence the total amount of noise generated is increased though the fundamental is of the same order of magnitude as in the low-incidence case. The sharp wake produces substantial increases in the high harmonics, and in the case worked the second harmonic dominated by 10 db.

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On the Coupling between Orthogonal Couette and Pressure Flows

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In the problem of leakage around high-speed shafts and also short journal bearings, a weak pressure flow acts at right angles to a strong Couette flow. For the case where the motion is turbulent, Tao[1] has been able to solve the short-bearing problem by neglecting any coupling effects between the two component flows. Constantinescu[2] has written mixing-length equations for such flows, also assuming the components to be independent. On the other hand, Prandtl[3] provided a mixing-length equation which permits accounting for coupling effects. The purpose of this note is to see what this predicts and to examine the restrictions involved in its application.

The mixing-length equation may be written in Cartesian coordinates as

$$T_{xy} = (\rho l^2 J + \mu)[(\partial u/\partial y) + (\partial v/\partial x)] \tag{1}$$

where J is defined by

$$J^2 = 2[(\partial u/\partial x)^2 + (\partial v/\partial y)^2 + (\partial w/\partial z)^2] + [(\partial w/\partial y) + (\partial v/\partial z)]^2 + [(\partial u/\partial z) + (\partial w/\partial x)]^2 + [(\partial v/\partial x) + (\partial u/\partial y)]^2 \tag{2}$$

Simplifying to apply to parallel flow (which is uniform in the sense that $\partial u/\partial x = \partial w/\partial y = \partial v/\partial z \equiv 0$), this becomes

$$T_{xy} = [\rho l^2 \sqrt{(\partial u/\partial y)^2 + (\partial w/\partial z)^2} + \mu](\partial u/\partial y) \tag{3}$$

Interchanging coordinates,

$$T_{zy} = [\rho l^2 \sqrt{(\partial w/\partial z)^2 + (\partial u/\partial y)^2} + \mu](\partial w/\partial y) \tag{4}$$

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Here the x and z coordinates are parallel to the boundary surfaces, and the y coordinate is normal to these. The velocity u in the x-direction represents the Couette flow, and the velocity w in the z-direction represents the pressure flow. The fluid density is ρ, the viscosity is μ, and the mixing length is l.

When τ_{zy} is the wall stress of the pressure flow, it is possible to write

$$\tau_{zy}(1 - y/b) = \rho[l^2 + \Delta(l^2)] \times \{(\partial u/\partial y)^2[(\partial w/\partial y)/(\partial u/\partial z)] + \mu(\partial u/\partial y)[(\partial w/\partial y)/(\partial u/\partial y)]\} \tag{5}$$

If it is assumed that $\Delta(l^2)/l^2$ is of the same order or smaller than w/u, and if $w/u \ll 1$, this simplifies to

$$\tau_{zy}(\partial u/\partial y)[1 - (y/b)] = \tau_{zy} w \tag{6}$$

where b is half the distance separating the plates. To permit integration, a Couette-flow profile must be used. Since it has been[4] shown that the profile $u/u_b = (y/b)^{1/7}$ fits experimental data, and since small departures are smoothed out by the integration process, Eq. (6) may be reduced to

$$T_{zy} u[1 - (1/8)(y/b)] = T_{xy} w \tag{7}$$

Applying this at a point halfway between plates where $y = b$, this becomes

$$T_{zy}/T_{xy} = (8/7)(w_b/u_b) \tag{8}$$

It is of interest that, if the z-direction flow had been an incremental Couette flow, the quantity 8/7 in Eq. (7) would be replaced by 1. Thus, it is seen that the resistance relative to a given wall is not greatly affected by the two types of pressure distribution.

Quite aside from questions concerning the validity of Eqs. (1) and (2), note that: (a) The solution did not require that viscous effects be neglected. (b) It was not necessary to assume an equation for mixing-length variation. (c) It was not necessary to assume that the mixing-length function remains constant, so long as the order of $\Delta(l^2)/l^2$ is the same or smaller—i.e., higher-order—than w/u, for the case where $w/u \ll 1$. As simplified in Eq. (4), the basic equation used is equivalent to stating that the eddy viscosity of the primary flow also serves as the eddy viscosity of the incremental flow. Furthermore, in Eq. (4) it is seen that the eddy viscosity ϵ is determined by the vector derivative of the combined velocity with respect to y; hence,

$$\epsilon = \rho l^2 \sqrt{^2(\partial u/\partial y)^2 + (\partial w/\partial z)^2} \tag{9}$$

As used, however, it has not been necessary to accept this relationship wholly. One need only accept that, in whatever true equation governs, the relative participation of the $\partial w/\partial y$ is such that it tends to vanish as $\partial w/\partial y \to 0$.

On the basis of the above arguments, one is tempted to con clude that for Eq. (8) to be a valid approximation, only a very low degree of validity is required of the equations which went into its derivation.

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